1a.

This is an AR(1) model where the stationary condition () is not met as . Therefore it is not covariance stationary.

1b.

This is an ARMA(1,1) model where . This model is covariance stationary if

In this case so this model is covariance stationary.

1c.

The first axiom of if something is covariance stationary is that E() is constant however in this case E() = and is therefore time dependant and not constant. Thus it is not covariance stationary.

2.

is an AR(1) model and the stationary condition is met, therefore it is covariance stationary.

The expected value is calculated as follows.

E() = E() + = =

We can see that this pattern will repeat infinitely and therefore,

E() = = =

Thus statement B is the only true statement.

3a.

The conditional mean of is the mean of given its previous terms. The previous terms are irrelevant however as E( ) = and thus the conditional mean, E( ) = 0.

3b.

4a.

To find the existence of a unit root we can use the Dicky Fuller test. When looking at an AR(1)

A unit root is present if

Using this test we can put and in the form of an AR(1) and look at the value.

=

By substitution we get that

Thus so has a unit root.

By substitution we get that

Thus so has a unit root.

4b.

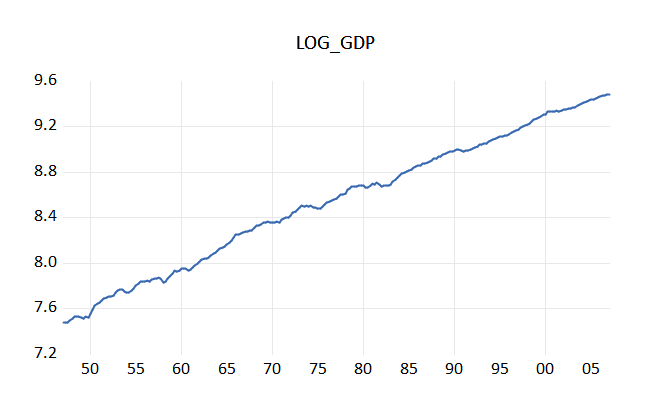
Cointegration is the process of looking at a combination of non-stationary variables to see if it yields something stationary, this must be linear combination.

For and we can see that which is covariance stationary so the cointegrating factors are 1 and -2.

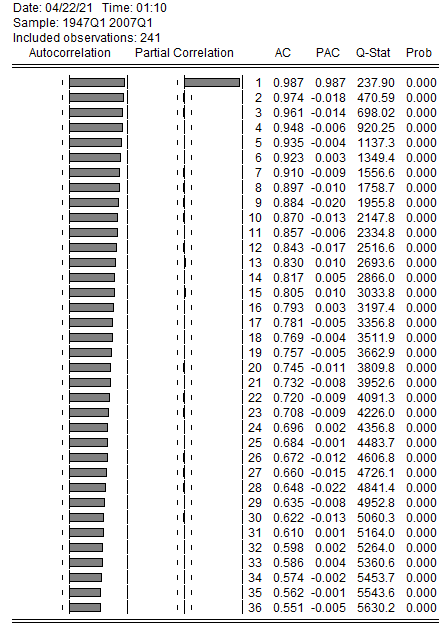
For and we can see that which is also covariance stationary with cointegrating factors of 1 and -2.

4c.

5a.



5b.



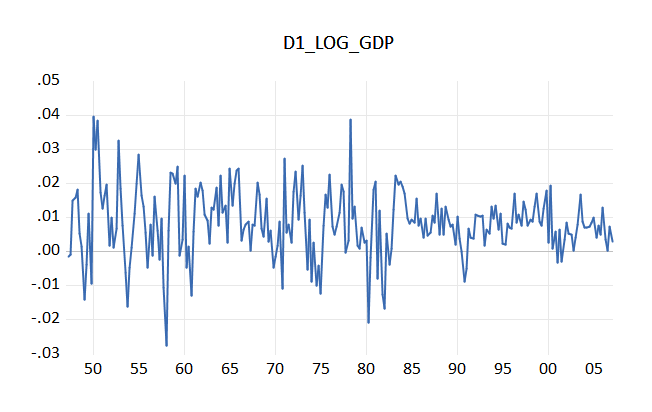
It is an AR(1) model where

5c.

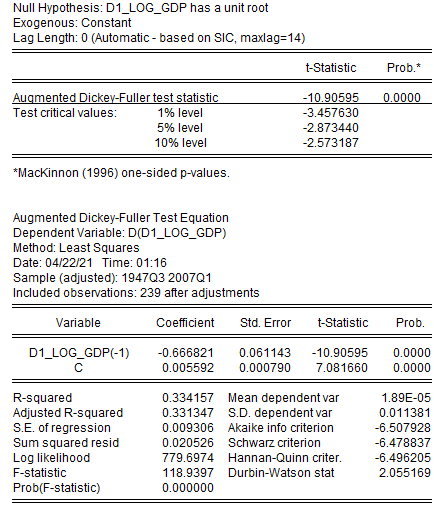
We can follow the 7 step Box-Jenkins process to estimate and forecast the “best: ARMA model(s)

STEP 1 – Apply data transformation if necessary. Test for stationarity/nonstationary. If the series is nonstationary, transform the data to achieve stationarity.

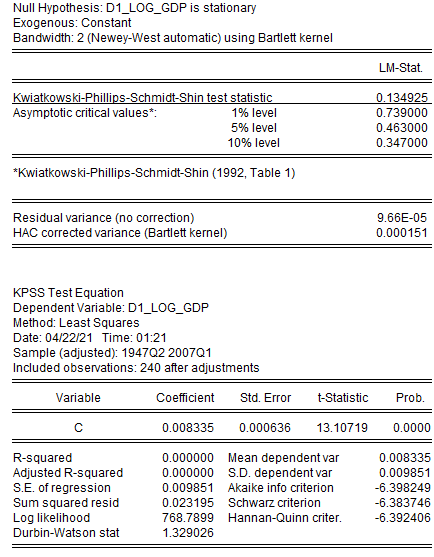
As shown by part a, log(gdp) is nonstationary so we may apply the transformation of the 1st difference.



This appears to be stationary, so we now test it with the ADF and KPSS.

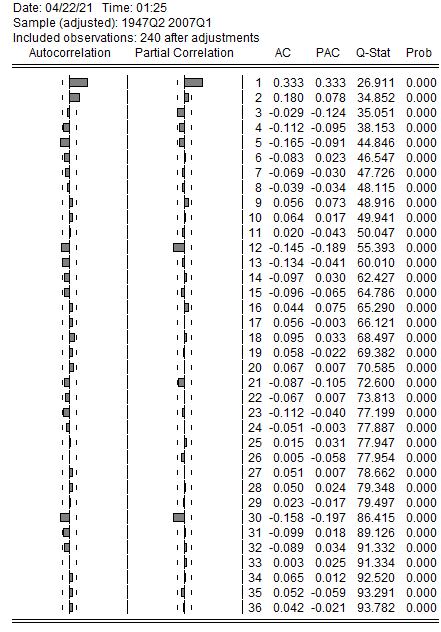


ADF shows us that it is stationary as the absolute value of the test-statistic is greater than the absolute value of the critical value at a 5% level, therefore we reject the null hypothesis



KPSS also shows us that it is stationary as the absolute value of the LM-Statistic is less than the absolute value of the critical value at a 5% level.

STEP 2 – Determine possible lag orders p and q in the ARMA(p,q) using (PACF/ACF)



From this we can drive values for p using Partial Correlation and q using Autocorrelation. Looking at the patterns in the ACF and PACF a model of ARMA(p,q) seems most suitable as all other models do not fit the shape of the functions.

Using this model P = (0,1) and Q = (0,1,2) leading to 5 possible combinations shown in the table below with their AIC and SBC.

|  |  |  |
| --- | --- | --- |
| ARMA(p,q) | AIC | SBC |
| ARMA(0,1) | -6.472133 | -6.428625 |
| ARMA(0,2) | -6.425215 | -6.381707 |
| ARMA(1,0) | -6.499140 | -6.455632 |
| ARMA(1,1) | -6.494217 | -6.436206 |
| ARMA(1,2) | -6.506747 | -6.448736 |

6a.

6b.

6c.